Construction of Virtual Trees within Ecosystems with EcoMod Tool

Aleš Zamuda, Janez Brest, Nikola Guid and Viljem Žumer Faculty of Electrical Engineering and Computer Science, University of Maribor Smetanova 17, 2000 Maribor, Slovenia Email: ales.zamuda@uni-mb.si

Abstract—In this paper we propose an interactive user friendly tool to design tree models. Our tool enables more flexible and rapid construction of procedural models by use of graphs to design local branch parameters as vector data. Our obtained procedural models can also be shaped by placing them in natural environments. Built procedural models can be used in up to perleaf precision animation for simulation of afforestation.

I. Introduction

For realistic computer visualization of natural environments it is mandatory to include certain level of vegetation (e.g. grass, trees, shrubs). For credible three dimensional rendering, geometrical descriptions of models are necessary, especially when we want to animate e.g. forest growth. We shall constrain this paper to modelling of trees, which are the most visible individual elements among vegetation in natural environments.

Several techniques for geometry model description and creation exist today. Because modelling of trees by hand is time consuming, procedural models techniques are usually used to create geometry. Procedural models base on various means to build main tree branching structure. They differ mainly in level of detail, for which they are applicable for, model creation rapidity, flexibility, ease of use, time and space requirements for model representation, possibilities for animation and final model representation method.

Aono and Kunii [1] developed one of the first specialized models for generation of trees. They created four geometry based models, each being more detailed model of the former. All models have some common rules. The first rule is that every branching of base branch forms two sub-branches. Length and diameter of branches diminish with an constant factor in direction from trunk to smaller branches. Branching angles are equal for all branches in same level. Plane on which sub-branches lie is perpendicular to the plane between base branch and precedent base branch. Branchings are always carried out on top of existing branches. In their model they also described the angle of axes rotation of successive branches, which is named *phyllotaxis* and dictates most of geometrical structure [2]. They also used vertex attractors which attached branches in directions of attractors for wind flow impact, sun rays and gravity.

Bloomenthal [3] built its model based on user administered skeleton. His model was only able to add saddles to branches and wrap up the branches with NURBS surfaces and textures.

Reeves and Blau [4] used particle systems to represent forest areas with lower detailed models. Branches were represented with straight lines, leaves with vertices or small circles and base trunks with cones. Parameters of their models were bottom width of crown, height of base trunk, average branch length and branching angle. Branching structure obtained was subsequently treated with algorithms for simulation of gravity, wind flow, gravity impact, and attraction to light.

Oppenheimer [5] modelled his trees using fractal techniques. Base trunk and branches were built by application of linear transformations defined by 3×3 matrices. Tree geometry consisted of prisms or straight lines with textures. These models were rendered for high-detail usage, but did not include any leaves.

Prusinkiewicz [6] created trees using biologically motivated models with L-systems [7] and they were applicable for high detail usage. He introduced visual representation of rewriting strings into L-systems, which were composed using context sensitive grammars. Visualization was similar to LOGO turtle [8] principle which was drawing geometrical elements of a string, representing topology of the tree. This technique was very flexible and several improvements were made with it. The down side of this technique is pretentious realization of a good modeller and therefore also the modelling itself, which requires user to know the domain specific language for definition of grammars for rewrite rules. Interactive modellers are emphasized in recent years for this technique [9], [10].

Holton [11] created trees using biologically inspired strand model. Thickness of branches and proportions between branching angles were determined directly with internal rules in the model. Strands flew along branches and divided without splitting a single strand. Branches with single strands were carrying leaves. Strand distribution determined branch thickness and their lengths. User entered number of strands along tree, proportions between branch lengths and branching angles to parametrize the procedural model. Certain attractors influenced the branching structure, e.g. central trunk uprightness, gravimorphism, phototropism, planartropism and phyllotaxis. Upside of this model was the mentioned automatic calculation for thickness of branches and certain proportions. A downside was that user still had to enter a huge amount of numerical data which diminished the flexibility of the model.

Weber and Penn [12] represented the tree model using simple geometry without development of branching topology.

For all branches in same levels they entered branching angle, branch length proportions and thickness for branches. They presented wind sway animation, branch cutting to predetermined volume, and progressive level of detail rendering.

Strnad [13] represented trees at middle level of detail with fractal hypertextures. Trees were defined using fractal iterative functional systems which were visualized using volume rendering. Because of its time-consuming rendering technique, his model was not applicable for real-time animation.

All listed techniques require a compromise between flexibility and ease of use. Some general purpose 3D modellers include specialized tools for construction of trees. Problem of these tools is that they are usually not flexible enough to model any tree model and usually allow modelling of only predefined families of tree species. If they are flexible, they are usually too flexible and do not allow user to enter data graphically but only numerically and the parameters are poorly described.

Our goal has been to develop a well-understood, simple and user friendly tool to design tree models. As trees can easier be compared to one another, we enabled a parallel construction of trees in ecosystems where simultaneous models can be shaped. Result of our work is a modeller for natural trees, which is integrated in a system for animation and simulation of forests.

The paper is divided in six sections. In the following section our procedural model is described. The third section lists parameters for two obtained procedural models and how they were obtained. In the fourth section rendering process of the whole ecosystem landscape is presented. The paper concludes with the summary and proposals for future work.

II. PROCEDURAL GEOMETRICAL TREE MODEL

The geometrical 3D model of a tree can be shaped interactively. By immediate image update each time some parameter is changed. The underlying procedural model also helps designing a tree from a minimized set of parameters user must set by automatically determining positions, rotations, sizes and textures for several thousand branch segments and several thousand leaves. Each branch and each leaf can be animated in real time to show growth of a tree or sway of a tree in the wind. The tree models designed with the tree modeller can be as foliage as also coniferous trees with very different branching structures.

Individual tree species model is created by parametrizing the procedural model. Parameters are distinguished as vectors (local) and scalars (global) and they are all designed interactively. Global parameters are constant for all branch segments but local parameters vary along (g, w) branch order.

Graphs help design vector parameters for strand distribution, branching angles, branch segment proportions, and gravity impact to tree geometry. Scalar parameters of the model are height and thickness of base trunk, wind impact, and density and size of leaves. Using listed vector and scalar parameters, geometrical model is built recursively. From an procedural model for a tree, a geometry model is calculated using the briefly denoted Algorithm 1. Geometrical model is rendered using photo textures for final look of a tree.

Algorithm 1 Calculation of geometrical structure of the procedural model tree. Recursive procedure is called using **branchsegment**(0, 0, S, 1, $l_0^{0,0}$, **I**, **I**, **I**).

procedure branchsegment(g, w, S_0 , L_0 , l_0 , \mathbf{M}_0 , $\mathbf{M}_{m:0}^{-1}$, $\mathbf{M}_{w:0}^{-1}$) **Require:** g, w - Gravelius and Weibull index of base branch; S_0 - number of strands in base branch; L_0, l_0 - base branch relative and actual length; \mathbf{M}_0 - base branch coordinate system; $\mathbf{M}_{m;0}^{-1}$ - inverse matrix of rotations for gravimorphism in coordinate system for base branch; $\mathbf{M}_{w;0}^{-1}$ - inverse matrix of rotations for directed wind in coordinate system for base branch; global k_d , k_c , l_{type} , $k_s^{g,w}$, $M^{g,w}$, $m^{g,w}$, $k_l^{g,w}$, $\alpha_m^{g,w}$, $\alpha_m^{g,w}$, t_c , w_g (see page 3) Ensure: rendered tree $d := k_d \sqrt{S_0}$; {thickness calculation from Mandelbrot} render base branch(\mathbf{M}_0 , l_0 , d); if $S_0 = 1$ then render leaves(l_{type}); **return**; end if $S_1 := \left\lceil 1 + k_s^{g,w} \left(S_0 - 2 \right) \right\rceil$, $S_2 = S_0 - S_1$; {number of strands in major and minor sub-branches} $r_1 := \max \left\{ \min \left\{ \sqrt{\frac{S_1}{S_0}}, M^{g,w} \right\}, m^{g,w} \right\}$ {branch length proportions dependant on strands} $r_2 := \max \left\{ \min \left\{ \sqrt{\frac{S_2}{S_0}}, M^{g,w} \right\}, m^{g,w} \right\};$ $L_1 := r_1 L_0, L_2 := r_2 L_0;$ {relative length of sub-branches} $l_1 := k_l^{g,w} L_1, l_2 := k_l^{g,w} L_2;$ {active sub-branch length} $\alpha_1:=k_c\sqrt{\tfrac{S_2}{S_0}}\alpha^{g,w},\,\alpha_2:=\alpha^{g,w}-\alpha_1;\,\{\text{branching angles}\}$ $\alpha_x(t) := \sin(t + R_x)w_s(1 - k_f)l_0$; {animation of un-directed wind $\alpha_z(t) := \sin(t + R_z) w_s (1 - k_f) l_0;$ $\alpha_w := \frac{S_0}{S} w_g$; {animation of directed wind impact} $\mathbf{M}_{1} := \mathbf{R}_{\mathbf{w}_{0}}(\alpha_{w})\mathbf{R}_{z}(\alpha_{1} + \alpha_{z}(t))\mathbf{R}_{x}(\alpha_{x}(t))\mathbf{R}_{y}(\alpha_{p})\mathbf{R}_{\mathbf{y}\times\mathbf{y}_{m}}$ $\mathbf{M}_{1} := \mathbf{R}_{\mathbf{W}_{0}}(\alpha_{w})\mathbf{R}_{z}(\alpha_{1} + \alpha_{z}(t))\mathbf{R}_{x}(\alpha_{x}(t))\mathbf{R}_{y}(\alpha_{p})\mathbf{R}_{\mathbf{y}\times\mathbf{y}_{m}}$ $(\alpha_{m}^{g,w})\mathbf{T}_{y}(l_{0})\mathbf{M}_{0};$ $\mathbf{M}_{2} := \mathbf{R}_{\mathbf{W}_{0}}(\alpha_{w})\mathbf{R}_{z}(\alpha_{2} + \alpha_{z}(t))\mathbf{R}_{x}(\alpha_{x}(t))\mathbf{R}_{y}(\alpha_{p})\mathbf{R}_{\mathbf{y}\times\mathbf{y}_{m}}$ $(\alpha_{m}^{g,w})\mathbf{T}_{y}(l_{0})\mathbf{M}_{0};$ $\mathbf{M}_{1}^{-1} := \mathbf{R}_{\mathbf{w}_{0}}(-\alpha_{y}^{g,w})\mathbf{R}_{z}(-\alpha_{y$ $\mathbf{R}_{\mathbf{y} \times \mathbf{y}_m}(-\alpha_m^{g,w})\mathbf{R}_y(-\alpha_p)\mathbf{R}_x(-\alpha_x(t))\mathbf{R}_z(-\alpha_1$ $\alpha_z(t))\mathbf{M}_{m;0}^{-1};$ {refreshing inverse matrix for construction gravimorphism vector, without considering wind impact} $\mathbf{M}_{m;2}^{-1}$:= $\mathbf{R}_{\mathbf{y}\times\mathbf{y}_m}(-\alpha_m^{g,w})\mathbf{R}_y(-\alpha_p)\mathbf{R}_x(-\alpha_x(t))\mathbf{R}_z(-\alpha_2)$ $\alpha_z(t))\mathbf{M}_{m;0}^{-1};$ $\mathbf{M}_{w;1}^{-1} := \mathbf{R}_{\mathbf{y} \times \mathbf{y}_m} (-\alpha_m^{g,w}) \mathbf{R}_y (-\alpha_p) \mathbf{R}_x (-\alpha_x(t)) \mathbf{R}_z (-\alpha_1 - \alpha_z(t)) \mathbf{R}_{\mathbf{w}_0} (-\alpha_w) \mathbf{M}_{w;0}^{-1};$ {refreshing inverse matrix for construction of disorded wind ways of the second state of the seco directed wind vector} $\mathbf{R}_{\mathbf{y}\times\mathbf{y}_m}(-\alpha_m^{g,w})\mathbf{R}_y(-\alpha_p)\mathbf{R}_x(-\alpha_x(t))\mathbf{R}_z(-\alpha_2$ $\alpha_z(t))\mathbf{R}_{\mathbf{w}_0}(-\alpha_w)\mathbf{M}_{w;0}^{-1};$ branchsegment $(g+1, w+1, S_2, L_2, l_2, \mathbf{M}_2, \mathbf{M}_{m;2}^{-1}, \mathbf{M}_{w;2}^{-1});$ {minor branch development}

A. Global parameters

development}

Global parameters of the procedural tree model (e.g. size of leaves) and initial parameters (e.g. total number of strands) are entered interactively, using a dialog window (Fig. 1). Data can be entered numerically or by changing sliders with bounds:

branchsegment $(g, w+1, S_1, L_1, l_1, \mathbf{M}_1, \mathbf{M}_{m;1}^{-1}, \mathbf{M}_{w;1}^{-1}); \{\text{major branch}\}$

- \bullet the number of strands S between 0 and 5000,
- the height of base trunk $l_0^{0,0}$ between 0 m and 10 m,
- the coefficient of branch thickness k_d between 0 and 0.05,
- the gravicentralism k_c between 0 and 1,
- the phyllotaxis angle α_p between 0° and 360° ,
- the density of leaves ρ_l between 0 and 30,
- ullet the size of leaves l_l between 0 and 0.3, and
- the speed of (un)directed wind w_q , w_s between 0 and 10.

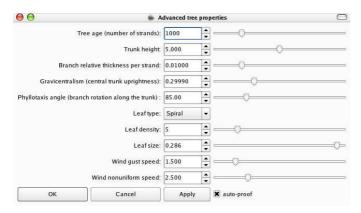


Fig. 1. Dialog window for design of global parameters for beech tree.

In the same window the leaf distribution l_{type} is also selected between following choices: *Spiral*, *Stacked*, *Staggered*, *Bunched* and *Coniferous*.

B. Local parameters

Branch order (g, w) dependant parameters are calculated using following equations:

$$k_s^{g,w} = \max\left\{\min\left\{k_s^g k_s^w, 1\right\}, \frac{1}{2}\right\},$$
 (1)

where $k_s^{g,w}\in [\frac{1}{2},1]$, auxiliary parameters are $k_s^g\in [\frac{1}{2},~1]$ and $k_s^w\in [0,2],$

$$\alpha^{g,w} = \min\left\{\alpha^g \alpha^w, 180^\circ\right\},\tag{2}$$

where $\alpha^{g,w} \in [0^{\circ}, 180^{\circ}]$, auxiliary parameters are $\alpha^g \in [0^{\circ}, 180^{\circ}]$ and $\alpha^w \in [0, 2]$,

$$\alpha_m^{g,w} = \max \left\{ \min \left\{ \alpha_m^g \alpha_m^w, 180^\circ \right\}, -180^\circ \right\},$$
 (3)

where $\alpha_m^{g,w} \in [-180^\circ, 180^\circ]$, auxiliary parameters are $\alpha_m^g \in [-180^\circ, 180^\circ]$ and $\alpha_m^w \in [0, 2]$,

$$M^{g,w} = M^g M^w, (4)$$

where $M^{g,w} \in [0,20]$, auxiliary parameters are $M^g \in [0,10]$ and $M^w \in [0,2]$,

$$k_l^{g,w} = k_l^g k_l^w, (5)$$

where $k_l^{g,w} \in [0,20]$, auxiliary parameters are $k_l^g \in [0,10]$ and $k_l^w \in [0,2]$,

$$m^{g,w} = m^g m^w. (6)$$

where $m^{g,w} \in [0,20]$, auxiliary parameters are $m^g \in [0,10]$ and $m^w \in [0,2]$.

Values of mentioned local auxiliary parameters are specified by defining control points of poly-line for graph. Their design is graphical and an example will be briefly presented in the following section. New control points can be easily added to poly-line and new line segments appear by that. By moving these control points, auxiliary parameter values on ordinate are changed according to abscissa for Gravelius (g) and Weibull (w) order. Even though both orders are integer numbers, we did not fixate positions of control points on abscissa with any means, because this would diminish flexibility of our



Fig. 2. Visualization of instanced geometrical models from single modelled procedural model for beech tree.

modelling. Visual hint helps when shaping graphs by colouring the changing part of the tree model so that user always knows what is being changed.

III. OBTAINED PROCEDURAL MODELS

Using our modeller, we were able to obtain several different procedural models for trees. Two obtained procedural models for two tree species follow.

A. Beech tree

To model the beech tree on Fig. 2 interactive design windows were set as shown in sub-figures of Fig. 3. These graphs have been internally converted to following procedural model parameters. The scalar parameters were: $S=1000,\ L_0=5,\ k_d=0.01,\ \alpha_p=85,\ k_c=0.3,\ l_l=0.2862,\ \rho_l=5,\ l_{type}=Spiral,\ w_g=1.5,\ \mathbf{w}=\begin{bmatrix}0&1&0\end{bmatrix}^T,\ k_f=1$ and $l_{LOD}=0.$

The branch distribution $k_s^{g,w}$ is determined for all orders (g,w), where $g\in[0,15],\ w\in[0,50]$ (g,w) are nonnegative whole numbers), with Eq. (1). The values k_s^g in k_s^w are calculated from both graphs, that are given with poly-lines $k_s^g=\{(0,0.75),(15,0.9)\}$ and $k_s^w=\{(0,1),(50,1)\}.$

The branching angle between dividing sub-branches, $\alpha^{g,w}$, is determined for all orders (g,w) using Eq. (2). Needed parameters in mentioned equation α^g in α^w are defined using two graphs, given as the poly-lines $\alpha^g = \{(0,45^\circ),(15,45^\circ)\}$ and $\alpha^w = \{(0,1),(50,1)\}$.

The minimum relative branch length, $m^{g,w}$, is determined for all orders (g,w) using Eq. (3), substituting $m^g=\{(\,0,1),\,(7.5,0.45),\,(15,1)\,\}$ and $m^w=\{(\,0,1),\,(50,1)\,\}$.

The maximum relative branch length, $M^{g,w}$, is determined for all orders (g,w) using Eq. (4), substituting $M^g=\{(\,0,1),\,(15,1)\,\}$ and $M^w=\{(\,0,1),\,(50,1)\,\}$.

The branch length scaling factor, $k_l^{g,w}$, is determined for all orders (g,w) using Eq. (5), substituting $k_l^g=\{(\,0,1),\,(15,1)\,\}$ and $k_l^w=\{(\,0,1),\,(50,1)\,\}$.

The gravimorphism impact, $\alpha_m^{g,w}$, is determined for all orders (g,w) using Eq. (6), substituting $\alpha_m^g=\{(0,0^\circ),(15,0^\circ)\}$ and $\alpha_m^w=\{(0,1),(50,1)\}$.

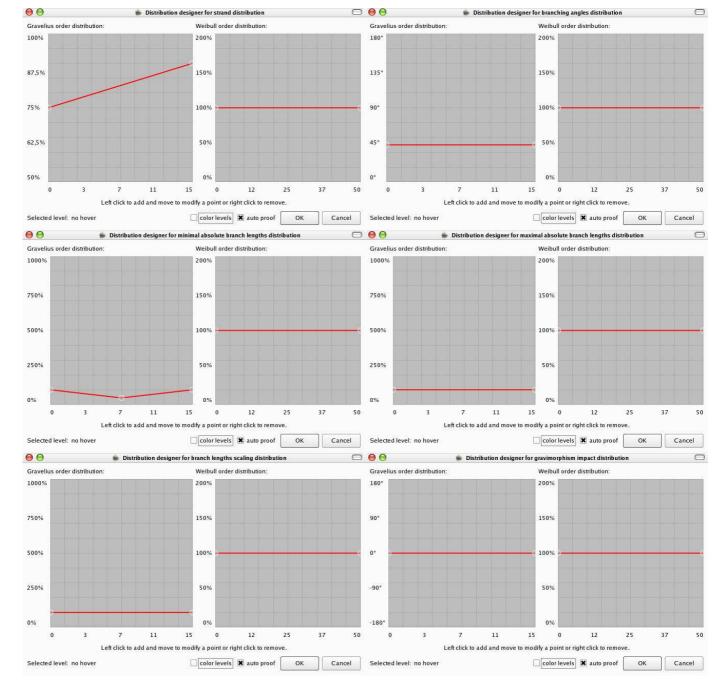


Fig. 3. Dialog windows for design of distribution graphs for beech tree: a) strands, b) branching angles, c) minimal absolute branch length, d) maximal absolute branch length, e) branch length scaling and f) gravimorphism impact.

B. Pine tree

To model the pine tree on Fig. 4 we have obtained following scalar parameters: S = 6000, $L_0 = 5.47$, $k_d = 0.005$, $\alpha_p = 85$, $k_c = 0.98$, $l_l = 0.1815$, $\rho_l = 30$, $l_{type} = \textit{Stacked}$, $w_g = 1.5$, $\mathbf{w} = [0\ 1\ 0]^T$, $k_f = 1$ and $l_{LOD} = 0$.

The branch distribution $k_s^{g,w}$ is determined for all orders (g,w), where $g \in [0,15]$, $w \in [0,50]$, with Eq. (1). The values k_s^g and k_s^w are calculated from both graphs, that are given with the poly-lines $k_s^g = \{(0,0.955), (0.7749,0.9615),$

 $(1.827, 0.8288), (4.207, 0.8269), (15, 0.8294) \}$ and $k_s^w = \{(0,1), (42.89, 1.005), (42.89, 0.654), (50, 0.6256) \}.$

Branching angle between the dividing sub-branches, $\alpha^{g,w}$, is determined for all orders (g,w) using Eq. (2). Needed parameters in mentioned equation α^g in α^w are defined using two graphs, given as the poly-lines $\alpha^g = \{(0,128.8^\circ), (1.164,17.91^\circ), (3.621,45.21^\circ), (15,35.83^\circ)\}$ and $\alpha^w = \{(0,1), (50,1)\}$.

The minimum relative branch length, $m^{g,w}$, is determined for all orders (g,w) using Eq. (3), substituting $m^g =$

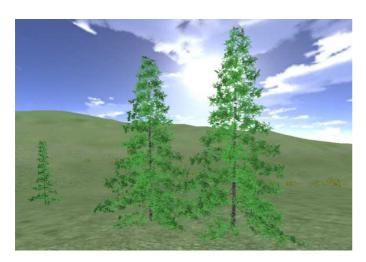


Fig. 4. Visualization of instanced geometrical models from a single procedural model for pine tree.



Fig. 5. Visualization using procedural models for living conditions inspired simulation of ecosystem.

 $\{(\,0,0.7109),\,(1.293,0.8531),\,(15,1)\,\}$ and $m^w=\{(\,0,1),\,(50,1)\,\}.$

The maximum relative branch length, $M^{g,w}$, is determined for all orders (g,w) using Eq. (4), substituting $M^g = \{(0,0.9953), (15,1)\}$ and $M^w = \{(0,1), (50,1)\}$.

The branch length scaling factor, $k_l^{g,w}$, is determined for all orders (g,w) using Eq. (5), substituting $k_l^g = \{(0,0.7109), (0.7749,0.5385), (2.522,1.327), (15,1)\}$ and $k_l^w = \{(0,1.014), (38.58,1.014), (45.26,0.5877), (50,0.01896)\}.$

The gravimorphism impact, $\alpha_m^{g,w}$, is determined for all orders (g,w) using Eq. (6), substituting $\alpha_m^g=\{(0,0^\circ),(0.3875,34.62^\circ),(1.034,-4.265^\circ),(1.55,-4.154^\circ),(2.974,-5.972^\circ),(15,-0.8532^\circ)\}$ and $\alpha_m^w=\{(0,1),(50,1)\}.$

IV. RENDERING OF ECOLOGICAL ENVIRONMENT

The composed procedural models can also be used to visualize natural environments. Natural environments can be created using simulation [14], [15], based on living conditions [16]. The living conditions we considered to build such environment

are height above sea level, slope, moisture, windiness and sunniness. A single inconvenient living condition can fully stop the prosperity of some tree. Fig. 5 shows an afforestation of an initially empty landscape across 6 km² of terrain with shrubs, beeches, pines, and maples. Mainly, land is populated with beeches, but several maples and few pines grow between them

V. CONCLUSION

In the paper we described our modelling tool for natural trees construction. By design of graphs flexible and rapid creation of procedural models was enabled. These models can be used to visualize landscape afforestation. The interdisciplinary research topic allows the use of developed algorithms on areas such as biology, ecology, forestry, and pedology. There are still many open research issues in this area. Our visualization module could be directly connected to a ray-tracer grid of computers for real-time rendering.

ACKNOWLEDGMENT

The authors would like thank to Jovan Hadži Institute of Biology and Geodetski zavod Slovenije, d.d. for support.

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